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Critical Tension: Sufficiency and Parsimony in QCA

Adrian Duşa[1]

Abstract

The main objective of the qualitative comparative analysis is to find solutions that display sufficient configurations of causal conditions leading to the presence of an outcome. These solutions should be less complex than the original observed configurations, as parsimonious as possible, without sacrificing the sufficiency requirement. Sufficiency and parsimony are two requirements that act in opposition, and an optimal solution is one that accommodates both. There are different search strategies that lead to different types of solutions, with an ongoing debate about which solution type is closest to the true, underlying causal structure. This article presents the different logics behind each simplification system in order to explain how and why they lead to different results and introduces the concept of "robust sufficiency" to clear the debate. It analyses the correctness ratios for the different solution type and provides an improved set of procedures to measure correctness that captures the best features from each system. Out of the competition between the conservative and the parsimonious search strategies, the intermediate solution emerges as the best hybrid that is suitable for causal analysis, outperforming the parsimonious solution in recovering a known (even parsimonious) causal structure.

[1] Department of Sociology, University of Bucharest, Romania

Corresponding Author:
Adrian Duşa, Department of Sociology, University of Bucharest, Şoseaua Panduri nr.90-92, Sector 5, 050663 Bucharest, Romania.
Email: dusa.adrian@unibuc.ro

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Introduction

If the sole purpose in qualitative comparative analysis (QCA) was to find causally relevant conditions, as argued by Baumgartner and Thiem (2017, hereafter B&T), their paper would be a game changer. But ever since its beginning, the main focus in QCA was to find minimally sufficient solutions that lead to the occurrence of an outcome.

Minimality is currently achieved by iteratively eliminating irrelevant causal conditions, employing a technique that originates from electrical engineering called Boolean minimization, which made its way to the social sciences through the groundbreaking work of Ragin (1987), the creator of QCA. The core minimization algorithm is called Quine–McCluskey (QMC), initially introduced by Quine (1955) and later on extended by McCluskey (1956).

Their original purpose was to obtain a minimal combination of inputs (closed and opened gates) that produce a certain electrical output. That purpose was preserved entirely alike in QCA: The outcome *has* to occur when associated with the simplified, minimized solution.

B&T attempt a formal response to a series of critiques over the correctness of the QCA solutions (among others, Hug 2013; Lucas and Szatrowski 2014; Munck 2016; Seawright 2014; Tanner 2014), using a systematic method involving an a priori known causal structure and test whether the different QCA solutions adequately recover that structure. While their strategy is sound, at least one problematic aspect is their recommendation that researchers " . . . should immediately discontinue employing the method's conservative and intermediate search strategy."

They declare both of these two solution types as faulty because they contain irrelevant factors in the causal mix and claim that the parsimonious solution is the only correct one because it is always free from irrelevancies.

In response to their findings, this article reveals that their conclusions are based on a simplification system that is unsuitable for causal analysis, stemming from a different type of logic called material implication. This system does an excellent job at identifying causally relevant factors but it incorrectly presents those factors as individually sufficient.

As it will be unfolded, all search strategies are bound to commit certain causal fallacies: The classical minimization sometimes does not manage to eliminate all irrelevant factors, while the system promoted by B&T sometimes presents insufficient conditions as sufficient. But their approach commits the worse type of causal fallacy, departing from the original purpose in QCA since it offers no guarantee that the outcome occurs. In addition, the parsimonious solution can be obtained by employing impossible counterfactuals, leading to logical impossibilities regarding sufficiency.

The next section presents several counter examples to uncover situations when the parsimonious solution fails to properly identify a known, underlying causal structure. To explain why and how that happens, and to describe the fundamental difference between the two simplification systems, a distinction will be made between the concepts of sufficiency and robust sufficiency. Finally, questioning B&T's definition of causal correctness, this article introduces some improved and more realistic procedures to measure the performance of all solution types, which captures the best features from both systems.

Counter Examples

The first example in this section is taken from Zhang (2017), whose setup is very similar to the simulations performed by B&T: a known causal structure, and test which of the QCA solution types correctly recover that structure.[1] The example is called PROF and describes an exam situation with two professors A and B and a student C. The formal rule in this situation is that *both* professors need to give a pass verdict for the student to pass the exam. In addition, we know that professor B is deferential to professor A, always peaks at the verdict and does the same (if A passes, B always passes and if A fails, B always fails). The known causal structure is therefore AB $\Rightarrow$ C.

Given professor B's deferential attitude toward professor A, we can only observe that both professors pass or fail a student, and it never happens that one of them pass and the other one fails a student. The observed data are therefore reduced at the very simple two lines from Table 1.

Analyzing these data with QCA, there are two possible solutions types to consider:

- The conservative solution (QCA-CS) that correctly recovers the true causal structure: AB $\Rightarrow$ C.
- The parsimonious solution (QCA-PS) that fails to recover this structure, stating the student passes if either of the professors (individually) give a pass verdict, with two models: A $\Rightarrow$ C, and B $\Rightarrow$ C.

Table 1. Zhang's PROF Example.

A	B	C
1	1	1
0	0	0

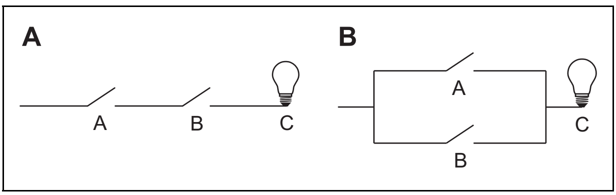

Figure 1. Alternative graphical representations of the two solution types. (A) Serial circuits A and B (qualitative comparative analysis–conservative solution). (B) Parallel circuits A or B (qualitative comparative analysis–parsimonious solution).

Clearly, these two solutions are not equivalent and they cannot be both correct. Since we already know the true causal structure from the initial setup of the situation, the conservative solution is the correct one.

This example can also be interpreted in terms of electrical circuits, with A and B representing two switches, and C as a light bulb. The conservative solution states that both switches are needed to turn on the light, while the parsimonious solution states that either of the switches A or B can independently turn on the light.

These two solution types are graphically represented in Figure 1, which captures the essential difference: When both switches are needed (as in the conservative solution), the circuit must be serial, and when either of them independently turns on the light (as in the parsimonious solution), the circuit must be parallel.[2]

From the observed data (in the usual notation where 1 represents the presence and 0 the absence), we notice that when the light is on, both switches are active, and when the light is off, neither of them is active. The observed empirical data only consist of those two situations, with no other information about the state of the light bulb when one switch is active and the other one is not.

Proponents of the parsimonious solution claim the two circuits are equally likely because they both reflect the observed data. After all, it can be just a coincidence that both switches are simultaneously on and off, with incompletely observed data. In order to accept the conservative solution as the true

Table 2. Complete Truth Table With Impossible Remainders.

A	B	C
1	1	1
0	0	0
1	0	?
0	1	?

Table 3. Subdata for the Causal Relation Between A and B.

A	B
1	1
0	0

causal configuration, we need to actually test (hence observe) that neither of the switches can independently turn on the light, otherwise we can assume that the parallel circuit (the parsimonious solution) is also plausible. And since this is always free of redundancies, it is considered superior.

Table 2 is similar to Table 1, with the two other (unobserved) configurations added below the dashed line. Short of actually observing these configurations in the absence of the light, the parsimonious solution is claimed to be equally likely.[3]

A simple solution to this situation can be found in Baumgartner's own work. He created a novel methodological approach called coincidence analysis (CNA; Baumgartner, 2009), which is specifically designed for causal analysis. There are some differences between CNA and QCA, but the main one is the built-in capability of CNA to treat each causal condition from the data as an outcome and search for possible causal configurations among the other columns.

There should be little debate that A and B are the only two possible causes, as it would be illogical to consider the light bulb as the cause of the switch or the student passing the exam as the cause of the professors' verdicts. For the subdata presented in Table 3, both the conservative and the parsimonious solutions agree that A is causally related to B: Whenever A is present, B is also present, and whenever A is absent, B is also absent.

There is no other empirical evidence to disrupt this pattern (A present and B absent, or A absent and B present), concluding that A is necessary and sufficient for B (confirming Zhang's experimental setup, A $\Leftrightarrow$ B), with coinciding conditions A and B.

From the empirical subdata in Table 3, we could conclude that A can be the cause of B, and equally likely that B can be the cause of A. Knowing that B has a deferential attitude, it is in fact possible to conclude that A is the cause of B, but that is less important. What matters is the empirical observation that when A $\Leftrightarrow$ B happens, then A and B either occur together or they are both absent.

This is an important observation and further implies two other points:

- Since A is causally related to B, it is impossible to observe one in the absence of the other, qualifying the additional unobserved configurations from Table 2 as impossible counterfactuals.
- Causal configurations that cannot occur (they are impossible) can be excluded from the Boolean minimization process, leading to an intermediate solution which, for this particular example, is identical to the conservative solution.

These two points imply a single logical conclusion that the intermediate/conservative solution is the true causal representation of these data and the circuit must be serial (professor A's verdict directly influences professor B's verdict). By implication, the parsimonious solution is incorrect and the circuit cannot possibly be parallel.

Defenders of the parsimonious solution might argue the expression AB $\Rightarrow$ C is logically equivalent to the expression A $\Rightarrow$ (B $\Rightarrow$ C) by virtue of *modus ponens* as demonstrated by Moisil (1969:537). Although this argument is logically correct, it misses an important point: The parsimonious solution B $\Rightarrow$ C explicitly uses remainders in the simplification process, and some of these remainders are impossible.

Resorting to the *modus ponens* argument can lead to logical impossibilities as presented in the second counter example from Table 4. Suppose now that condition A denotes pregnancy, condition B a female, and the outcome C represents extremely safe driving, under the plausible statement that *pregnant females drive extremely safe* (naturally, both A and B are conjunctively needed to comply with the sufficiency statement).

These data are similar to an example from Schneider and Wagemann (2012), where their example had drunk driving as an outcome, and it extends Zhang's example by an additional observed configuration that nonpregnant

Table 4. Relation Between Pregnancy (A), Females (B), and Extremely Safe Driving (C).

A	B	C
1	1	1
0	0	0
0	1	0
- - -	- - -	- - -
1	0	?

females do not drive equally safe. In this example, however, it should be fairly clear that being a woman is not sufficient for pregnancy, while being pregnant is sufficient to know the person is a female. This leaves a single remainder configuration under the dashed line, a truly impossible pregnant man which logically cannot be proved sufficient for the outcome.

Yet again, the conservative solution manages to recover the correct causal structure AB $\Rightarrow$ C, while the parsimonious solution states that pregnancy alone is sufficient for extremely safe driving: A $\Rightarrow$ C.

Using the inverse Boolean minimization rules, A $\Rightarrow$ C is equivalent to the expression $A(B + \neg B) \Rightarrow C$, which is further equivalent to $AB + A \neg B \Rightarrow C$. This in turn means that $A \neg B \Rightarrow C$, leading to the logical impossibility that $A \neg B$ (the pregnant man) is both sufficient and insufficient at the same time. Such impossible configurations should be excluded from the minimization process, leading to an intermediate solution.

These are counter examples with proven demonstrations that contradict the results presented by B&T, who claim the parsimonious solution QCA-PS is always correct. At least two questions should be addressed in the next sections:

(a) Why does QCA-PS present an insufficient solution as sufficient?
(b) How is it possible for the parsimonious solution to be 100 percent correct, yet failing to identify true, known causal structures?

Robust Sufficiency or Parsimony?

Before delving more deeply into matters of how the different simplification systems achieve sufficiency, it is important to clarify the meaning of "causal analysis." As a direct response to their paper, this article adopts the same interpretation of causal analysis as B&T's, who employed the regularity theory of causation as discussed by Mackie (1974), who in turn introduced the concept of Insufficient but Necessary part of an Unnecessary but Sufficient (INUS) condition.

Through his very definition of the INUS condition, Mackie specifies that, in order to qualify as a cause, a configuration must be *sufficient* for the outcome: Whenever the cause is present, the outcome will also be present. Sufficiency might not be enough for causal inference under the regularity theory, but it is a relevant ingredient.

An INUS condition is an essential part of a sufficient causal configuration,[4] but it is insufficient by itself. Only the whole conjunction is sufficient, and it loses this property once any of the relevant INUS conditions are taken out. When a certain outcome is caused by a combination of relevant (nonredundant) factors, none of them are individually able to make the outcome occur. In both examples from the previous section, neither A nor B alone can individually instantiate the outcome, but they are conjunctively needed to achieve sufficiency: $AB \Rightarrow C$.

The trouble with sufficiency is that it has two different interpretations that lead to different methods of achieving parsimony. It is therefore important to clarify how each simplification system defines sufficiency and attempt to differentiate between them.

The classical method (Boolean minimization) achieves sufficiency by comparing all pairs of observed configurations to determine which pair differs by only one literal (that can be minimized) and iteratively performs this process until nothing further can be minimized to obtain the so-called prime implicants (the solution terms).

Vital, in this very short description, is that QMC works by default with the *observed* positive configurations only. The truth function can be further simplified to a more parsimonious solution through counterfactual reasoning, using unobserved configurations of causal conditions (remainders in QCA or *don't care*'s in electrical engineering circuits) as if they were conducive to the presence of the outcome.[5]

Table 5 is the simplest possible example that illustrates how the Boolean minimization operates: Two causal conditions A and B are distributed over two observed instances, both of them displaying the presence of an outcome Y. In the first instance, both A and B are present, and in the second instance, A is present and B is absent. The following is an equivalent DNF (disjunctive normal form) expression: $AB + A \neg B \Rightarrow Y$.

By applying the distributive law from Boolean algebra, the condition A is found as a common factor for both B and $\neg B$, which cancel each other out: $A(B + \neg B) \Rightarrow Y$. Thus, the causal condition B is eliminated as irrelevant

Table 5. Empirically Observed Cases.

A	B	Y
1	1	1
0	0	1

Table 6. Truth Table for the Material Implication.

A	Y	A ⇒ Y
F	F	T
F	T	T
T	F	F
T	T	T

Note: T = true; F = false.

for the outcome Y, leaving condition A as the only, minimally sufficient one: A ⇒ Y.

The very same conclusion can be reached by applying a very different procedure stemming from the classical propositional logic, where a particular logical function is called *material implication*: A ⇒ Y. This can be translated into natural language as "if A, then Y" (sometimes also "A only if Y"), and it has the same interpretation as the sufficiency statement[6]: A ⇒ Y.

Material implication has a certain property with respect to its truth value (Hurley 2012:325), adapted and displayed in Table 6. The first two columns describe all possible logical values between the antecedent (A) and the consequent (Y), and the third column contains the truth value of their sufficiency relation. Here, all values are true (including the first two lines where the antecedent is false) except for the third row, where A is true and Y is false. The only possible way to invalidate the sufficiency relation between A and Y is to observe a true value for A, combined with a false value for Y. For all other situations, the sufficiency relation cannot be logically rejected.

Using the data from Table 5, the sufficiency statement A ⇒ Y can be tested using the material implication. On the first line, a value of 1 (true, presence) for A is associated with a value of 1 for Y, corresponding to the fourth line in Table 6 that confirms the truth value of the sufficiency statement. On the second line, a value of 0 (false, absence) for A is associated with a value of 0 for Y, corresponding to the first line on Table 6 that cannot reject the truth value of the sufficiency statement.

As there is no empirical evidence in Table 5 to associate the presence of A with the absence of Y, the sufficiency relation is considered theoretically possible. Using the same propositional logic, both B and $\neg$ B are also identified as sufficient for Y, but they cancel each other out. This technique confirms the Boolean minimization process that identified A $\Rightarrow$ Y as the only, minimally sufficient condition.

Since both systems arrive at the same result, and the property of material implication is much more efficient,[7] then why did Quine chose to develop an algorithm based on a highly expensive Boolean minimization? He could not have overlooked the application of propositional logic on this type of problems, as he was well aware of its properties and even used it for a solution to Hempel's well-known raven paradox (Quine 1969).

A possible explanation for his decision can be found in Popper's (1959) theory falsification. He rejected the inductive approach for scientific discovery, showing that a particular statement such as "This swan is white" cannot be extended to the universal statement that "All swans are white." The universal statement can be analyzed using again the Table 6:

- Observing anything that is not a swan (either white or not white) does not falsify the statement.
- Observing a white swan surely does not falsify the statement (if anything, it can only confirm it) but is still unable to prove the universal statement as true.
- The only possible way to falsify the statement is to actually observe a swan that is not white.

For quite some time, it has been believed that being a swan is sufficient to be white. As long as scientists did not observe a swan that was not white, they could not falsify the universal statement, but soon enough the Australian scientists actually observed a black swan and proved the inductive theory as false: Being a swan is not automatically sufficient to be white.

Material implication works well for theory falsification but not for causal analysis: The fact that we currently observe anything but a falsifying case is by no means a formal proof for the truth value of the sufficiency statement. A theory stands "true" until it is falsified, but despite the fact that "not false" is equivalent to "true" in Boolean logic, being unable to falsify a theory cannot be a sufficient proof for its confirmation.

Table 7. A Scientifically Realistic Truth Table for Material Implication.

A	Y	$A \Rightarrow Y$
F	F	$\neg$ F
F	T	$\neg$ F
T	F	F
T	T	$\neg$ F

The only indicative conditional that has a truth value is: true $\Rightarrow$ false, which is always false, but no other such conditional has a truth value. The misleading "true" should be replaced with a more realistic "not false" as presented in Table 7.

It follows that material implication involves an ambiguous trivalent logic: false, true, and not false. In Boolean algebra, not false is equivalent to true, but with material implication it has a different meaning and cannot not be associated with a truth value.

This discussion was necessary to reveal there are different definitions of the same concept of sufficiency, which seems to float in ambiguity. To differentiate between them, I am using the concept of "robust sufficiency,"[8] with the following definition:

Definition 1: A disjunct in a QCA solution is robustly sufficient if the outcome is guaranteed to occur in its presence.

In formal notation, a disjunct Φ in a QCA solution for an outcome O is robustly sufficient for O if and only if:

- Φ is sufficient for O.
- No proper part of Φ is sufficient for O.
- O is guaranteed to happen when Φ is present.

Boolean minimization guarantees the outcome will always occur, although it cannot guarantee the solution is parsimonious enough and free from irrelevancies. On the other hand, material implication guarantees the solution is the absolute most parsimonious, but it cannot guarantee the outcome always occurs. Parsimony and robust sufficiency are both important but they create a critical tension in opposite directions: Striving for robust sufficiency might not eliminate redundant conditions from the causal configuration, while striving for parsimony might sacrifice the robust sufficiency.

Out of these two requirements, however, robust sufficiency should prevail. Researchers should seek parsimony to the largest possible extent, but the first priority is the occurrence of the outcome. Under this new perspective, B&T's correctness ratios will be revisited in the next section, to assess which of the different solution types best describe the true, underlying causal structure.

Correctness Ratios

The purpose of QCA is to find minimal causal configurations that are robustly sufficient for the *presence* of an outcome. Especially for the purpose of causal inference, for any given outcome at least one causal path[9] must be instantiated.

In their attempt to answer various critiques over the robustness of QCA solutions, the correctness ratios presented by B&T strike as unlikely, and sometimes even impossible. Given a saturated truth table, a natural expectation is to have decreasing correctness ratios from 100 percent (upper left corner) for the complete data to 0 percent (lower right corner) when all rows are deleted (zero diversity).

Instead, figure 4 from their paper displays decreasing slopes until 11 or 12 deleted rows, after which the correctness ratios spring back to life and reach an impossible 100 percent correctness for 0 percent diversity (all 16 rows deleted). The sudden rejuvenation of QCA-CS (conservative solution), QCA-IS_1 (intermediate solution with correct directional expectations), and QCA-IS_2 (intermediate solutions with misspecified directional expectations) is explained by their choice to include the empty set ($\varnothing$, no QCA solution) in the list of 24 preserving correctness models $M_{CC}(\Delta')$ from their figure 3.

While $\varnothing$ can be a logical subset of anything, including their initial causal configuration from model $m(\Delta')$, it is illogical to claim that *nothing causes something*.[10] By counting situations with no solution as correct, it is either that (at least) 1 of the 24 models from $M_{CC}(\Delta')$ (from here on, denoted by $M_{\mathbf{24}}(\Delta'))$ violates their own configurational correctness principles or their definition of configurational correctness is misspecified. As it will be shown in this section, both statements apply and the QCA-CS solution type is an ideal test ground to start an investigation.

Upon close inspection of these 24 models, an immediate observation becomes obvious: They range from the most complex model "w", that is, the data generating causal structure $m(\Delta')$, toward the simplest proper parts

Table 8. Identical Correctness Ratios for QCA-CS Under Different Model Spaces.

Deleted rows	$M_{24}(\Delta')$	$M_8(\Delta')$
1	43.750	43.750
2	16.667	16.667
3	5.536	5.536
4	1.648	1.648
5	0.504	0.504
6	0.212	0.212
7	0.149	0.149
8	0.163	0.163
9	0.219	0.219
10	0.262	0.262
11	0.252	0.252
12	0.385	0.385
13	1.786	1.786
14	8.333	8.333
15	31.250	31.250
16	100.000	100.000

(supersets) of all terms from this model such as $\neg$ A model "a", that is also a superset of the term $\neg$ AB model "e".

Table 8 presents the correctness ratios employing the 24 models in the first column, compared to a smaller search space containing the following eight models: $\varnothing$, $\neg$ AB, B $\neg$ C, D, $\neg$ AB $\vee$ B $\neg$ C, B $\neg$ C $\vee$ D, $\neg$ AB $\vee$ D, and $\neg$ AB $\vee$ B $\neg$ C $\vee$ D.

The empty set was intentionally retained in $\mathbf{M_8}(\Delta')$ to reveal a remarkable finding: The correctness ratios are identical to those from $\mathbf{M_{24}}(\Delta')$.[11]

What is, then, the purpose of the other 16 models? Actually, of the other 17 models, counting the empty set as incorrect? The answer is obvious, it was the only possible way to guarantee a tautological 100 percent correctness ratios for the parsimonious solution type, QCA-PS. It is therefore not a magical property of the parsimonious solution to recover the true causal structure, but rather a matter of what is defined and considered "correct."

Evaluating the correctness of the QCA-CS against $\mathbf{M_{24}}(\Delta')$ is a contradiction in terms, because the principles of Boolean minimization state that it is impossible for the conservative solution to become more parsimonious than the original model $m(\Delta')$. Quite the opposite: The lower the diversity, the more complex the conservative solution.

Besides, a proper causal analysis should find minimally and robustly sufficient terms, not individual INUS conditions from (or supersets of) these terms.[12] If this is true, then none of the other 17 models from $\mathbf{M_{24}}(\Delta')$, containing (insufficient) supersets of the true causal structure $m(\Delta')$, can be considered correct. This is actually confirmed by Mackie (1974:62) himself, whose work is heavily relied upon by B&T: “ABC is a *minimal* sufficient condition: [. . .] no part of it, such as AB, is itself sufficient for P.”

The correctness ratios presented by B&T are further questionable, given their curious choice of dealing with model ambiguity, even under the forced model space $\mathbf{M_{24}}(\Delta')$ where it is still impossible for the parsimonious solution to be 100 percent accurate. For instance, by deleting the third row from the saturated truth table, the QCA-PS contains two models: $\neg AB \vee B \neg C \vee D$, and $\neg AC \vee B \neg C \vee D$.

The first model is identical with the initial causal structure, while the second should have been declared incorrect (according to their own definition) since the term $\neg AC$ is not listed in $\mathbf{M_{24}}(\Delta')$. In order to maintain a 100 percent correctness ratio they used a second, theoretically unjustified decision to tweak the definition (p. 4): “ . . . it must be guaranteed that the latter is truthfully reflected by at least one presented model . . . .”

The “*at least*” part is problematic.[13] While in a real-life scenario it is impossible to determine which of the different models from a solution are correct, in this example having one incorrect model of two, the correctness ratio should be equal to 50 percent.[14]

These are several pointers that suggest they used an ad hoc definition for causal correctness, an obviously biased one given their uniform 100 percent correctness ratio series for the QCA-PS. In order to properly evaluate QCA, it is mandatory to agree on a bias-free definition. The remainder of this section attempts to answer the natural question: What is or what should be considered correct?

A first suggestion already exists in the QCA literature (see Ragin 2008; Ragin and Sonnett 2005), where it is well known that QCA-PS is a superset of the QCA-CS: They are extreme ends of the complexity–parsimony continuum, with both solution types covering the observed positive configurations from the truth table. This is also confirmed by Mackie (1974:299): “ . . . (i) All A are C. Therefore, All AB are C . . . .”

For instance, if two causal conditions M (taking medication) and W (staying warm) would constitute a conjunctive term in the sufficient model $MW \Rightarrow H$ (restore health from a cold), then any other more complex term such as MWC (where C could mean playing chess) is a subset of the parsimonious term MW. The doctors might prescribe the medication and indicate to stay out of the cold weather, as well as playing chess, even though MW would be sufficient to restore the health, chess being irrelevant.

A sufficient superset guarantees that all of its constituent subsets are also conjunctively sufficient (they are part of a sufficient superset, a free ride in a sufficient vehicle), therefore they should also be considered correct. A causal recipe containing irrelevant factors involves unnecessary actions (like playing chess or clap hands and sing a song), but it would never reduce the chances of reaching a positive result.

The purpose of any causal analysis being to find the conditions (or configuration of conditions) that are associated with the presence of the outcome (as mentioned in the Introduction section), for such a causal configuration to be correct the outcome must occur. Another way to combine Mackie's regularity theory of causation with the concept of correctness is to look at the problem from the other side and assess under which conditions a solution cannot be correct: In order to be consistent with the purpose of the causal analysis, *a solution*[15] *is incorrect if the outcome fails to occur in its presence* (i.e., it is not robustly sufficient).

In the hypothetical example, if both taking Medication and staying Warm are conjunctively needed to restore the Health outcome, and the parsimonious solution identifies either Medication or Warmth as individually sufficient (in a similar way like the PROF example), then Health will certainly not going to be restored.

By allowing such insufficient conjuncts to be counted as "correct," B&T introduced a confusion between minimality and parsimony in clear contradiction with Mackie (1974:61):

> ". . . All ABC are followed by P, but it is not the case that all AB are followed by P . . . ."

B&T's principles of configurational correctness (**CC**) should be revised, perhaps with a single statement that:

> **Definition 2:** A disjunct from a correct solution is a robustly sufficient subset of the true causal structure.

If the conservative solution is a subset of that structure, it is also correct in spite of containing irrelevant factors. To draw a parallel, it is similar to a very low resolution picture of an image, where the aim is to eliminate the noise from the picture by increasing the resolution, up to the point where the picture is identical with the image. The more irrelevant factors are

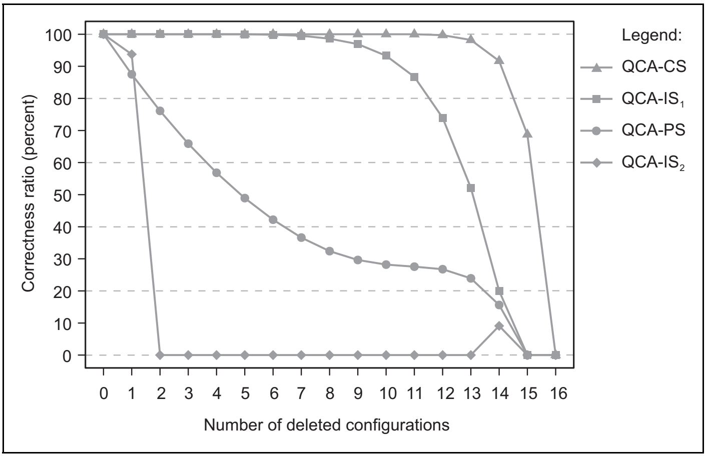

Figure 2. The real correctness ratios.

eliminated, the more resolution is added, and the solution becomes identical with the true causal structure.

This requires a serious amendment of their "causal fallacy" interpretation to consider a solution as correct if it does not commit any causal fallacy.[16] While the definition is logical and can be agreed upon, their operationalization is biased because they would reject MWC as causally effective (since C is not a proper part of a potential "true" causal structure MW).

In reality, researchers may simply not have enough observed data to eliminate C from the causal configuration, but with proper investment in both theory and data collection, this condition might be identified as irrelevant in the future.

A more realistic version of their series correctness ratios is presented in Figure 2, and tells a very different story.[17] Contrary to their version, the conservative solution is actually the best performing solution type. This happens because each term from this solution is a subset of a term from the parsimonious solution, up to 10 rows deleted from the saturated data. As the most complex solution type, the conservative solution is guaranteed to contain more causal conditions than necessary, but on the other hand, it is always robustly sufficient, as a subset of the true causal structure.

Perhaps unsurprisingly, the parsimonious solution (their best performing solution type) is actually the third performing correctness series, with a rapidly decreasing slope for lower diversity values, at a very large distance from the first two solution types. The worst performing solution type is

QCA-IS_2, the intermediate solution with all directional expectations formulated against theory, with a zero correctness ratio after only two deleted rows.

QCA-IS_1, the intermediate solution with proper theoretical directional expectations, performs almost as well as the conservative solution QCA-CS and explains the high interest around this solution type: It is usually less complex than the conservative solution (which means it manages to eliminate more redundant conditions), while at the same time maintaining a subset relation with the true causal structure.

While B&T allow insufficient (even atomic INUS) conditions as correct, the alternative definition 2 introduced above allows irrelevant conditions in conjunction with the causally relevant, correct ones. To satisfy the requirements from both systems, another possibility is to find those models that are as close as possible to the true causal model: *robustly sufficient* expressions that contain no irrelevant conditions.

Such a scenario can be tested by comparing the models from the solution types in the strictest possible way, with the true causal model itself. The previous configurational correctness definition could be revised as:

> **Definition 3:** A disjunct from a correct solution is a robustly sufficient subset of the true causal structure that commits no causal fallacies (no irrelevant and no insufficient conditions).

Given that all of the true model terms $\neg$AB, B$\neg$C, and D are causally relevant and robustly sufficient, following a configurational line of thought their conjunctions are also causally relevant (they contain no irrelevant conditions) and robustly sufficient[18]: $\neg$AB$\neg$C, $\neg$ABD, B$\neg$CD, and $\neg$AB$\neg$CD. Any superset disjunction formed by various combinations of these terms is considered correct.

Figure 3 displays the new correctness ratios under this new, more strict definition. As expected, the complex conservative solution containing irrelevant conditions has a much lower correctness series. Surprisingly however, at almost all levels of diversity the intermediate solution with proper directional expectations QCA-IS_1 constantly outperforms the parsimonious solution QCA-PS and emerge superior in recovering the true underlying model.

While the correctness ratios under definition 2 seem too relaxed, the series from Figure 3 might seem too strict, given they eliminate all models that contain anything but the original causal structure.

Another, final possibility to calculate the correctness ratios would be to make more justice to model ambiguity. I have previously shown that after deleting the third row from the saturated truth table, the parsimonious

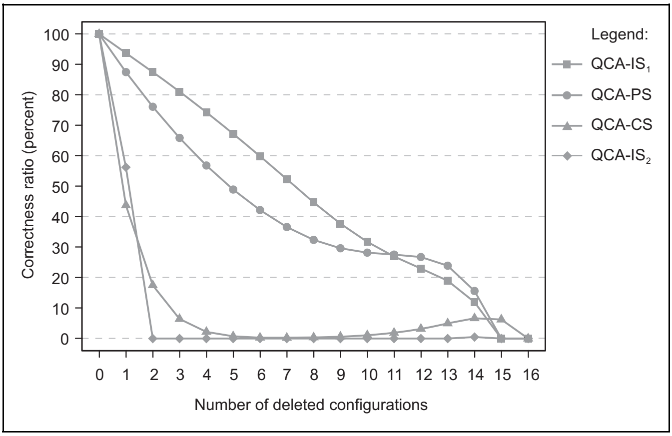

Figure 3. The series correctness ratios with strict complexity penalty.

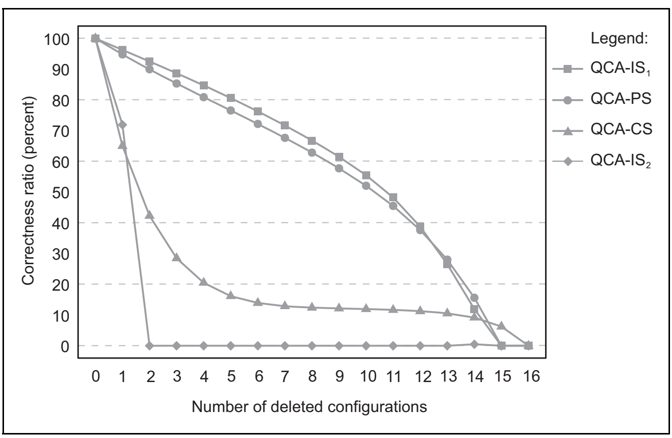

Figure 4. The series correctness ratios with relaxed complexity penalty.

solution QCA-PS contains two models: $\neg AB \vee B \neg C \vee D$ and $\neg AC \vee B \neg C \vee D$.

Under the strict rule, the second model is considered incorrect because it contains $\neg AC$, which is not present in their $M_{\mathbf{24}}(\Delta')$ space. However, the second model correctly identifies the disjunction $B \neg C \vee D$ from definition 3. With two correct terms of three, it could be considered as 66.7 percent correct, with an aggregated correctness ratio for the entire solution of 83.3 percent instead of 50 percent as identified under the strict rule (See Figure 4).

The overall idea is to give as much credit as possible to the parsimonious solution QCA-PS and to verify if any of the other solution types (especially QCA-IS_1) can perform at least at the same level.

Once again, albeit at a very small margin, the intermediate solution QCA-IS_1 with proper directional expectations consistently outperforms the parsimonious solution QCA-PS in recovering the true underlying causal structure, while the conservative solution QCA-CS performs marginally better.

In all test scenarios, the intermediate solution with misspecified directional expectations QCA-IS_2 has a zero correctness ratio after just two deleted rows. This is a testimony to the importance of correctly specifying directional expectations, crediting QCA-IS_1.

All these scenarios, however, used a rather limited definition of the intermediate solution, which in fact can be obtained through any conceivable method of excluding some of the remainders from the minimization process. Specifying directional expectations separates the so-called simplifying assumptions (a subcategory of remainders that actually contribute to parsimony) into two categories: easy and difficult counterfactuals, and the intermediate solution is obtained by excluding the difficult ones.

But the QCA methodology is far richer than that (Duşa 2019; Ragin 2008; Ragin and Sonnett 2005; Schneider and Wagemann 2012, 2013; Yamasaki and Rihoux 2009), with many other categories of remainders that should be excluded: logical impossibilities (such as the pregnant man), simultaneous subset relations (configurations with high consistency scores for both the presence of the outcome and its negation), contradictory simplifying assumptions (that help achieving sufficiency for both the outcome and its negation), and even a special category that combines the analysis of sufficiency with the analysis of necessity: Those incoherent counterfactuals for which the negation of a necessary condition is logically insufficient for the outcome.

The parsimonious solution is completely oblivious to all of these so-called untenable assumptions, and indiscriminately employs all of them in the simplification process. A proper solution should strike a balance between including those remainders which can be included and excluding those which should be excluded. With more such difficult, untenable, impossible remainders being blocked from contributing to parsimony (with proper theoretical justifications), the intermediate solution will get closer to the true underlying structure, with a higher likelihood of retaining robust sufficiency.

To mirror the results from B&T, all analyzed test scenarios exclude nothing else but the difficult counterfactuals, as a result of using directional expectations. However if the true causal model involves any other type of untenable assumptions, the correctness ratio for the intermediate solution is likely to increase even further.

It is beyond the scope of this article to dig more deeply into this matter, as it would be difficult to mirror B&T, but it is nevertheless important to mention that the intermediate solution can be further refined to bring it closer and closer to the true underlying causal structure, thus improving its correctness ratio.

Conclusions

Ever since its appearance, the methodology of QCA has been debated and has raised a fair number of critical papers. The initial criticism of being restricted to crisp sets led to the development of the fuzzy sets version. The more recent attention to solution robustness and generally to how close the QCA solutions are to the real causal mechanism pushes the methodology in a constant improvement cycle.

In response to many QCA critics, B&T's inverse search strategy starts from a known causal structure that is used to generate a saturated truth table, which is then used to assess how well the different solution types manage to recover that structure under different levels of diversity.

However sensible this strategy is, their conclusions seem unlikely because of a twofold choice of using an ad hoc definition of causal correctness, combined with the employment of propositional logic that is unsuitable for causal analysis. It should be applauded for identifying causally relevant INUS conditions, but the real purpose of QCA is to achieve robust sufficiency and to make sure the outcome occurs when associated with a causal configurational.

In their search for the optimal solution, researchers should strike the right balance between finding what the reality is versus what the reality is not, between detecting false negatives versus false positives, between "signal versus noise" (Ragin 2014:82), and between confirmatory power and disconfirmatory power, in a similar vein to the balance between type I (rejecting a true hypothesis) and type II (not rejecting a false hypothesis) errors from inferential statistics.

All of these are important, and a good solution should avoid being one sided. This article improves the procedure of measuring correctness and confirms what has been long suspected: A proper intermediate solution outperforms the parsimonious one in recovering a known causal structure and is positioned closest to the true, underlying causal model.

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ORCID iD

Adrian Duşa https://orcid.org/0000-0002-3525-9253

Supplemental Material

Supplementary material for this article is available online.

Notes

1. Zhang is critical to both coincidence analysis and qualitative comparative analysis (QCA) regarding the parsimonious solution, but he appears to have missed that QCA's other solution types are able to solve his examples.
2. Some may argue that a parallel circuit is exclusively equivalent to the function $A + B \Rightarrow Y$, and not to the alternative solutions $A \Rightarrow Y$ (or) $B \Rightarrow Y$. Both are possible, however, depending on the second step from the Boolean minimization

process to cover the prime implicants (PI) chart. In the PROF example, the correct solution is A $\Rightarrow$ Y or B $\Rightarrow$ Y because the PI chart has only one column, whereas a union type solution such as A + B $\Rightarrow$ Y needs at least two columns (two observed positive configurations).

3. Baumgartner and Thiem (hereafter B&T) claim it is actually superior because it does not commit any causal fallacy. Their interpretation of a causal fallacy is addressed later.
4. In this article, a causal configuration is usually equivalent to a conjunction of causal conditions.
5. In electrical engineering, the goal is to find the simplest possible (most parsimonious) equivalent function that minimizes the cost of a circuit, but other solution types are also possible. Subsequent algorithms have been developed by Duşa (2010; see also Duşa 2018; Duşa and Thiem 2015) that reach exactly the same results as the classical Quine–McCluskey without explicitly using remainders, an approach that is now called "pseudo-counterfactual."
6. The horseshoe $\supset$ notation should not be confused with the similar notation from set theory $\supset$, where it means a superset. In logics, $\supset$ is a reversed C letter that means: *is contained in* (which is the same thing as the expression: *is a subset of*).
7. A very similar algorithm has been advanced by Duşa (2018).
8. This concept appeared during the numerous discussions with Jiji Zhang, to whom I owe my gratitude not only for the concept but more importantly for his highly original perspective of modeling causal irrelevance, not just causal relevance.
9. A cause (or a causal path) can mean anything, from a single causal factor to any combination of multiple factors in a conjunctive and/or disjunctive form.
10. Some may argue that the empty set does not make any causal inference, and by consequence it is not incorrect. This is the same kind of logical application of material implication where $\neg$ False is not always equal to True, hence not being wrong cannot be counted as correct either.
11. Without the empty set, the ratios begin to differ as from 10 or more deleted rows, where there are situations when all positive configurations are deleted and no solution is possible.
12. B&T are right in saying that, for a true causal structure such as AB $\Rightarrow$ Y, in a complex conservative solution ABZ $\Rightarrow$ Y, only A and B are relevant insufficient but necessary part of an unnecessary but sufficient conditions and Z should be removed because it is causally irrelevant. Mackie does not say, however, that A and B are atomically (individually) sufficient as implied under their model space $M_{\mathbf{24}}(\Delta')$.
13. They justified their decision by misleadingly quoting Spirtes, Glymour, and Scheines (2000:81), but that particular page and in fact the entire book are

completely unrelated to model selection in QCA, let alone to the problematic "*at least*."
14. A possible counterargument is to blame the data, not the method. However, I believe it is impossible to derive a 100 percent correct conclusion based on a completely deficient data, despite the fact this is made possible by material implication.
15. By a "solution," I am actually referring to each and every disjunct of the solution.
16. This is actually the very principle of the Boolean minimization process, which iteratively eliminates all redundant factors until nothing further can be eliminated.
17. Here, correctness is calculated as a ratio of the models that are subsets of the true causal structure out of all possible models. Admittedly, many of the conservative solution models are proper subsets (hence correct), but at a very low resolution containing many irrelevant factors. This could be improved by calculating how well a solution performs as a subset, while at the same time how close it is to the true, underlying causal structure.
18. Any conjunction of robustly sufficient terms is bound to be robustly sufficient itself.

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Author Biography

Adrian Duşa is a full Professor in the Department of Sociology at the University of Bucharest, where he teaches statistics and reseach methodology, and the President of the Romanian Social Data Archive (RODA). He specialized in the Qualitative Comparative Analysis, having authored the QCA package in R, and published numerous articles and books on this topic. He also serves in the Steering Committee and Advisory Board of COMPASSS, the professional community dealing with Comparative Methods for Systemic Cross-Case Analysis.