Comparative analysis using Boolean algebra

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Abstract

Comparative analysis is an important methodological tool in the social sciences, a *via media* between qualitative and the quantitative research designs. It extends classical qualitative analysis through a systematic method of comparing cases, and complements quantitative data analyses through a novel approach involving a mathematical algorithm that employs Boolean algebra. When it is difficult to apply statistics due to a very small number of cases, comparative analysis can still uncover important causal patterns. Unlike regression-based techniques that rely on symmetric correlations, qualitative comparative analysis presents an alternative way to analyse social science data using set theory, by comparing all possible pairs of cases to determine which causal conditions are redundant (not associated with the outcome of interest) and which configurations of surviving causal conditions are minimally sufficient for the outcome. This methodology allows researchers to identify causal relevance from even a small number of cases, combining Boolean algebra with philosophical concepts such as sufficiency and necessity, as well as counterfactual analysis. Specific to Boolean algebra and Qualitative Comparative Analysis is a feature called equifinality, identifying multiple causal paths that lead to the same outcome.

Introduction

Unlike the natural sciences (especially the physical ones) where controlled experiments can be organized in dedicated laboratories, research in the social sciences is very different. Leaving aside that isolating all possible factors except for the experimental treatment would be practically impossible, it is ethically forbidden to apply mass social experiments.

The only possible way to understand the human, social life is through observation, comparing events for similarities and differences. Daniele Caramani (2009) made an excellent, historical overview of the comparative method, from John Stuart Mill's cannons to their application in social (and predominantly sociological) classical theory by Emile Durkheim and Max Weber.

Durkheim (1982) postulated that "comparative sociology is not a particular brand of sociology, it is sociology itself" (p.157). Indeed, everything in the social sciences is a matter of similarity and difference. The components of the social life start to make sense only when comparing units, for instance the same person, region or country can be either rich or poor depending on a reference point.

More complex situations appear when analysing causal relationships between different phenomena, when the infinite complexity of the social life makes it extremely difficult to pinpoint very specific effects of certain causal conditions over an outcome of interest. This is partly due to the difficulty of accurately measuring social phenomena, and partly because the effects combine and it is difficult to isolate individual influence. Weber's *ideal type* is a helpful methodological tool, making it possible to identify at least some combinations of factors that act together to instantiate the outcome.

While Weber used his ideal types in the context of what he referred to as mental experiments, more recently sociologist Charles Ragin (1987) found a novel way to use these ideal types in a systematic fashion, adapting an algorithm stemming from electrical engineering to the social sciences.

Ragin's comparative method uses the so-called truth tables, another methodological tool that is attributed to the German philosopher Ludwig Wittgenstein, later introduced in the social sciences by American sociologist Paul Lazarsfeld with his *attribute space*, or what is currently known as a *property space*.

As will be shown in the next sections, no instance is perfectly similar to the ideal, but they all cluster close to common configurations of ideal types. Instances differ from one another depending on where they are positioned in the property space: those which cluster in different configurations are said to be *different in kind* while those which cluster around the same configuration are said to be similar in kind but *different in degree*.

Two other comparative strategies have a similar logic to Mill's inductive canons, identified and used by Adam Przeworski and Henry Teune (1970):

- MSSD (most similar systems design) studies cases that are as similar as possible, assuming their high similarity increases the chances to find the factors responsible for their differences
- MDSD (most different systems design) studies cases that are as different as possible, to demonstrate the causal effect is strong enough under different conditions.

Both of these systems can be further combined on cases that display the same outcome, or on cases that display different outcomes, a key methodological technique that paved the way to Ragin's own contribution: qualitative comparative analysis (QCA).

Binary system and Boolean algebra

Dating back to the ancient Chinese terms of Yin and Yang that describe the duality of nature, the binary system made its way into Western culture around the 18th century through the work of philosopher and mathematician Gottfried Leibniz. His strong belief in the power of symbols for human understanding led him to invent a binary mathematics that used only two values: 1 and 0.

Leibniz devoted his entire life to this system, and his philosophy led him to believe it has divine origins, with mystical properties whereby 1 represents the good and 0 the evil. Without any concrete applications, he created transformation methods from base 10 to base 2, and even constructed a special machine that did that automatically. The academic community of his time ignored his work and its applications had to wait over 200 years until computers appeared and his binary system spread to an extent that only Leibniz could have dreamed of.

About 150 years after Leibniz, around the middle of the 19th century, another great mathematician named George Boole refined the binary system until it became useful for logics and mathematics. The academic community again ignored this work, with its first applications appearing decades later at the Massachusetts Institute of Technology (MIT) in the United States. Similar to Leibniz, the special algebra he created uses only two values (0 and 1), but Boole's contribution was to substitute these values for *false* and *true*.

The first real applications of the Boolean algebra, that had a dramatic effect on today's society, appeared in electrical engineering where truth values were adapted to closed and opened gates, leading to the modern computers that still operate in the binary system.

In its simplest form, Boolean algebra can be described with a series of three basic operations: conjunctions (logical AND), disjunctions (logical OR) and negations (logical NOT). Briefly presented

Table 1: Logical AND

A	B	$A{\cdot}B$
0	0	0
0	1	0
1	0	0
1	1	1

Table 2: Logical OR

A	B	$A + B$
0	0	0
0	1	1
1	0	1
1	1	1

Table 3: Logical NOT

A	$\sim A$
0	1
1	0

here, these are the building blocks of the modern algorithms that enable QCA, to solve highly complex research scenarios using the power of Boolean algebra.

The Boolean operations can be best described using Wittgenstein's truth tables, that display all possible combinations of antecedents and their truth value result, replacing 0 for *false* and 1 for *true*.

Conjunctions result in a true value only when all antecedents are true. For a result to happen, all conditions must be met—for instance scientific performance is a function of both intelligence and hard work. None of these individually generate performance, but in conjunction they lead to positive outcomes.

In Table 1, the only combination where the result Y is true is found on the fourth row, where both A and B are true. The conjunction is a is essentially a set intersection (calculated as the minimum between the values in A and the values in B), formally denoted by an equation such as: $A{\cdot}B \Rightarrow Y$.

Disjunctions result in a true value when any of the antecedents is true. Contrary to the conjunctions, it is always true unless all antecedents are false. Table 2 presents the same combinations of antecedents A and B, where Y is always equal to 1 (true) except for the first row where both A and B are false.

Disjunctions play a special role in comparative analysis. One and the same outcome can be produced by many potential causal configurations, a situation called *equifinality* or *multiple causation*. They are best described as a set union (calculated as the maximum between the values in A and the values in B), formally denoted with the + sign into an equation such as: $A + B \Rightarrow Y$.

Negations are the simplest of the Boolean operations, inverting the truth value of the antecedent: if the antecedent is false the result becomes true, and if the antecedent is true the result becomes false, as presented in Table 3. It is sometimes denoted by the $\neg$ sign, while other textbooks prefer using a tilde $\sim$ sign.

Sometimes, certain phenomena can be produced not by an active event, but rather quietly by its absence. In sociological theory, it is well known that societal anomie tends to grow in the absence of an established normative system.

All of these operations can be combined in any conceivable ways to produce an outcome, with various combinations of conjunctions and disjunctions of causal factors, each containing presence and / or absence of the factors.

Table 4: Boolean minimization example

A	B	Y
1	1	1
1	0	1

The logic of qualitative comparative methodology

In the beginning of electrical engineering, the circuits used in control and automated systems used to quickly grow in complexity with very many gates. The task was to obtain a circuit that was as simple as possible, eliminating redundant gates but preserving the output of the circuit intact (say, a lightbulb being lit).

This is remarkably similar to social science research, whereby multiple factors have a potential contribution in producing a certain event, with the same overall objective to eliminate redundant factors and preserve only those that are causally relevant in producing the event. Just like in electrical engineering, the goal is to obtain a causal configuration that is associated with some event being instantiated.

Maintaining the presence of the event is a key feature: it would be pointless to design an electrical circuit that does not output the required electrical signal, just as it is pointless to identify causally relevant factors if the phenomenon of interest does not happen.

The Boolean operations presented in the previous section are heavily employed in the comparative analysis methodology, with a direct application in what is called *Boolean minimization*, the most important instrument of eliminating irrelevant causal factors (from here on referred to as causal conditions, or simply conditions).

Boolean minimization searches for those causal conditions that are consistently associated with the presence of the outcome. If some phenomenon A is a truly relevant cause of another phenomenon Y, then it should always be present when the outcome happens.

Table 4 presents a situation where the outcome Y happens (it has a value of 1) in both instances, but the condition B is present in the first instance (case) and absent in the second. This is a typical scenario that eliminates condition B as causally irrelevant. Out of the two conditions, A is the only one that is consistently associated with the presence of the outcome Y.

This procedure is repeated for all possible pairs of cases, highlighting its comparative nature: for each pair of two cases, the algorithm has to determine if they differ by exactly one literal (condition), and if that happens the literal where they differ is eliminated to produce a so-called *implicant*. This procedure is iteratively repeated for all possible pairs of subsequently generated implicants, eliminating everything that can be eliminated, until reaching a solution that contains the smallest number of causal conditions that are conjunctively responsible with the presence of the outcome.

The higher the number of initial causal conditions, the more possible combinations of two cases will need to be verified. This makes the procedure not only extremely slow but also potentially reaching out of memory since the number of generated implicants can quickly grow towards infinity.

Alternative solutions have been found, with better and better algorithms that are faster and consume less and less memory by Adrian Dușa (2010), Dușa and Alrik Thiem (2015), and a more recent version called CCubes presented by Dușa (2018) where the final solutions are exactly the same, but with dramatic improvements in speed and memory management.

It is perhaps important to mention that relevant causal conditions should not (and actually cannot) be associated with the absence of the outcome. If some causal phenomenon A causes another phenomenon Y, it means that Y is always produced when A is triggered. In other words, it is impossible to have the cause but not the effect (to have A in the absence of the outcome Y): when the cause is present, the outcome will certainly be produced.

The resulting, minimal conjunctions of causal conditions are called *prime implicants* (or *solution terms*), and they can disjunctively combine in various ways to explain (to cover) the observed configurations where the outcome is present. A QCA solution is therefore a minimal disjunction of conjunctions of causal conditions that are associated with (sufficient for) the presence of the outcome.

This is just a summary presentation of the core minimization procedure behind Ragin's QCA. He managed to successfully adapt an algorithm designed for electrical circuits to social research, enabling the analysis of (sometimes qualitative) data that are free from the host of restrictions and especially assumptions that are so common in the quantitative, statistical tradition.

Methodologically, QCA belongs to a family called configurational comparative methods (CCM; Rihoux & Ragin, 2009), where complex cases should be transformed into configurations of causal conditions, in order to allow for *systematic* comparisons. These methods are specifically designed to work with small-N or intermediate-N (where statistical analysis is impossible to be applied), although it is a common misperception that QCA operates only at this level.

In reality, the number of cases is almost irrelevant as it can operate with any number of cases, starting from as few as two (the absolute minimum to have at least one comparison). What matters most is the number of unique causal configurations the cases gravitate around, something that is presented in the next sections.

Set theory

The binary system is well suited for the language of sets. Social science research is also embedded with sets, although not always explicit. Categories of nominal or ordinal variables (e.g. urban / rural, men / women) are very similar to sets: the set of people living in urban areas, the set of females, for example. Mathematics abounds in notions such as the set of real numbers, or the set of integer numbers etc. In short, a set can be defined as a collection of objects that share a common property.

Concerning social research in general, and for comparative analysis in particular, there are two main and formal classes of sets: the *crisp* family (binary crisp sets, and multi-value crisp sets) and the *fuzzy* sets.

In crisp sets, an element is either in or out of a set. A person is either living in a rural area, or not; is either living or not; either does something or not. Fuzzy sets, on the other hand, can have an infinite number of values with partial membership of elements to sets.

Ragin's original QCA book from 1987 exclusively treated the crisp version of sets, and researchers gradually understood that social life is too complex for such a crude dichotomization. A country does not simply belong to the set of democratic countries or not, but almost always has a partial inclusion in that set.

Answering this criticism, Ragin (2000; 2008) extended QCA methodology to what is currently known as *fuzzy sets QCA* (fsQCA). What is perhaps most innovative in this extension is the ability to reduce an infinite number of possible fuzzy inclusions in sets, to a finite number of ideal-types. He managed to preserve the classical approach to social research, but boosted the procedure with the newest mathematical advancements.

In formal notation, binary crisp sets can have only two values: 0 if the element does not belong to a set, and 1 if the element belongs to the set.

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \notin A \\ 1 & \text{if } x \in A \end{cases} \tag{1}$$

Unlike crisp sets, an element can have a fuzzy membership score to a set anywhere between 0 and 1 (Zadeh, 1965). The values 0 (full exclusion) and 1 (full inclusion) are just the extremes of an infinite continuum, where a value such as 0.1 signals a very small inclusion in the set, whereas 0.97 signals a very high (almost complete) inclusion. The value 0.5 is the point of maximum ambiguity in fuzzy set relations, and it is usually avoided by QCA practitioners.

Having the same interval of variation with probabilities, between 0 and 1, fuzzy sets are sometimes confused by quantitative researchers. The difference should be clear though, with a simple example. If there is a probability of only 1% that a stove is very hot, then there is a small (but definitely existing) chance to get severely burned by touching it. In fuzzy sets, if the stove has an inclusion of 0.01 in the set of very hot objects, there is absolutely no risk of getting burned by touching it.

The values 0 and 1 play a key role in comparative analysis, referring to ideal types. Continuing with the democracy example, no country in the world has a value of 0 (completely excluded from the set of democratic countries). Any country has at least some partial membership in this set, even if very small. To the other extreme, 1 refers to a complete inclusion in the set of democratic countries, and it should be stressed again that no country in the world has a complete inclusion in this set. Even in the most democratic countries, there are at least some undemocratic activities that prevents a perfect inclusion score.

Instead, the values of 0 and 1 refer to "ideal-typical" situations that do not exist in reality but researchers nevertheless use them to describe cases that are more or less near one of the extremes. This is precisely what Weber recommended when he developed his social science research methodology.

But set membership scores are not limited to only 0s and 1s. Multi-value crisp sets can have any number of unique whole numbers (referring to crisp intermediate states between 0 and 1), as proposed by Polish mathematician Jan Łukasiewicz (1970) and generalized to multi-valent systems with $n = v - 1$ values, obtained through a uniform division of the interval [0, 1]:

$$\mu_A(x) = \left\{ 0 = \frac{0}{n}, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n}{n} = 1 \right\} \tag{2}$$

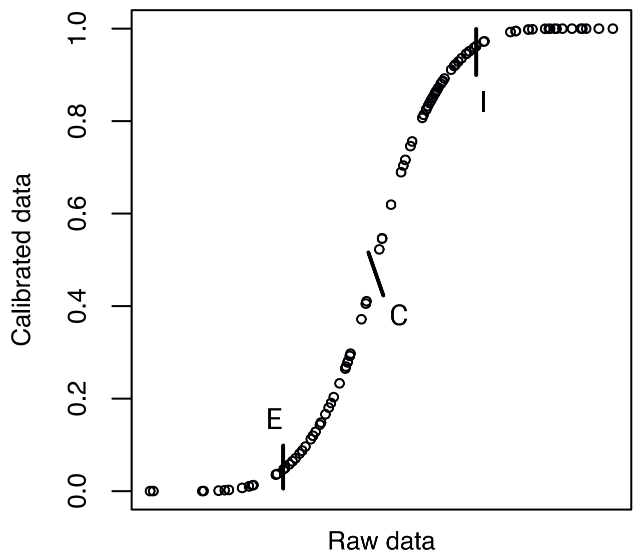

Figure 1: Typical fuzzy calibration using a logistic function

The calibration process

Operating in paired comparisons, the QCA algorithm requires that all data should be provided in set membership scores with all values ranging from 0 to 1. A key step in this comparative methodology is to coerce the raw data to fuzzy or crisp sets, a process called *calibration*. The input raw data are usually numeric, although there are some possibilities to transform categorical data into fuzzy values.

Calibrating numerical raw data to binary crisp sets is essentially a recoding procedure, specifying a certain threshold: values below are recoded to 0 and values above are recoded to 1. Recoding into multi-value crisp sets is just as simple, specifying two or more thresholds. The general procedure produces $x + 1$ values, where x is the number of thresholds: one threshold produces a two valued (binary) crisp condition, two thresholds produces a three valued crisp condition, and so on.

Calibrating into fuzzy sets is more challenging and requires a certain level of practical and theoretical expertise. Despite the precise mathematical procedures to achieve this type of calibration, it is far from a mechanical process since it involves a set of thresholds that are highly qualitative and theory dependent.

The most well known calibration procedure is called the *direct method*, described by Ragin (2000) and illustrated in Figure 1. The fine grained fuzzy membership scores follow a logistic function (with its specific s-shape) that needs a set of three thresholds: one to determine the full inclusion in the set (I), one to determine the full exclusion from the set (E) and the cross-over point of maximum ambiguity (C). The choice of thresholds should be an informed one, based on theory, and the thresholds should reflect unambiguous changes in the interpretation of the numerical data.

Ragin explains this process by appealing to the values of 0 degrees Celsius (at which water freezes) and 100 degrees Celsius (at which water boils). Compared to the millions of degrees at the surface of the Sun, a difference of 100 degrees seems unimportant but this precise temperature span reflects the two points where water changes its aggregation state: from liquid to solid and from liquid to

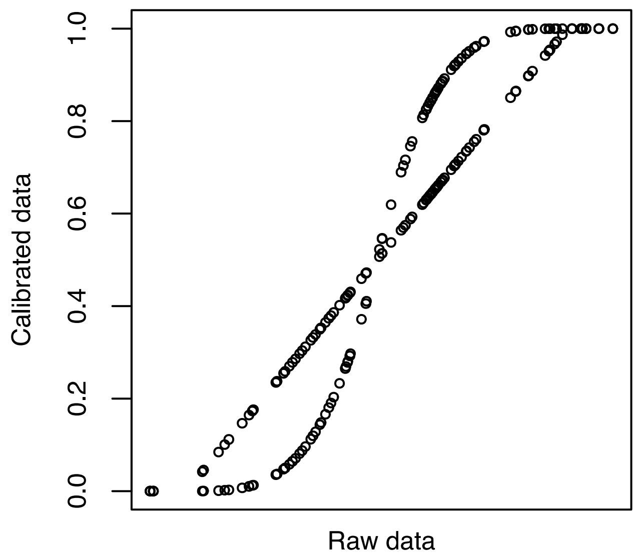

Figure 2: Comparison of linear and logistic calibration functions

gas. Despite their numerical values these are qualitative thresholds that are humanly interpretable, rather than having some intrinsic mathematical properties.

In the same vein, researchers should identify those numerical values at which their conditions change in quality. How high should gross domestic product (GDP) be to consider a country (surely) developed? How high should a democracy score be for a country to be considered fully included in the set of democratic countries? The same kind of questions refer to the opposite side, establishing thresholds for full exclusion from the set of developed countries, or from the set of democratic countries.

"Development" and "democracy" are complex concepts that can have multiple definitions, and based on how they are defined, the choice of thresholds can be different. A definition that holds in a certain country or culture (e.g. ethics, perceived discrimination) might be very different for another, especially if the definitions are specific to one nation or another.

Even for the same definition, in the same country, the thresholds' values can change in time. For instance referring to age in the mid 19th century, the meaning of "old" was very different than today. In some countries, the life expectancy at that time was about 40 years, whereas in the 21st century a person at the same age in Japan (with a life expectancy of more than 80 years) could be considered even young. Studies about social distance, as defined by Emory S. Bogardus, would likely yield different results almost a century later due to the increased diversity of races and ethnic groups in the present society.

A good practical advice in comparative studies is to always include methodological details regarding the calibration process for every causal condition included in the analysis, so that other researchers can replicate the results using the same set of thresholds.

The logistic function, although widely used due to its use in the fs/QCA software by Ragin and Sean Davey (2017), is only one way among others to obtain set membership values. Figure 2 presents an alternative, linear function with a monotonic increase from lower left (full exclusion) towards the upper right corner (full inclusion).

This particular linear function also needs a set of thresholds, as well as other parameters presented in Equation 3, and it is available in other software, particularly in the R programming environment as presented by Thiem and Dușa (2012), and Dușa (2019).

$$dm_x = \begin{cases} 0 & \text{if } x \leq e, \\ \frac{1}{2}\left(\frac{e-x}{e-c}\right)^b & \text{if } e < x \leq c, \\ 1 - \frac{1}{2}\left(\frac{i-x}{i-c}\right)^a & \text{if } c < x \leq i, \\ 1 & \text{if } x > i. \end{cases} \tag{3}$$

where:

- e is the threshold for full exclusion
- c is the crossover
- i is the threshold for full inclusion
- a and b determine the shape above and below the crossover

Such linear calibrations are capable of things outside the realm of the logistic function, for instance calibrating using a trapezoidal function (a bell-shape if smooth), assigning full membership in the middle and excluding very low and very high values. If researchers are interested to explain or to introduce in the explanation the set of medium developed countries, the bell shape functions are the only solution.

Categorical raw variables are also subject to calibration. For instance Nicolas Legewie (2017) proposes an interesting method to calibrate purely qualitative data using an anchored framework. While there is little to complain against the systematic method, it is questionable that a single categorical variable can capture the whole complexity of a social science concept.

While categories of nominal variables can be directly transformed into binary crisp sets (a process very similar to obtaining dummy variables in various regression analyses), more attention is needed with categories from ordinal variables (most notably, those with a Likert-type response scale). It is tempting to assume these variables follow an underlying continuous distribution and assign fuzzy membership scores to each category, a process called *direct assignment* (Verkuilen, 2005), which is a very weak and highly subjective type of calibration.

In general, if the number of ordered categories is small (up to 3 or 4) a solid possible strategy is to calibrate to a multi-value crisp set. When the number of categories is large, another possible solution is to use the so-called *totally fuzzy and relative* (TFR), using an empirical cumulative distribution function (E) and calculating the distance from each CDF value to the cumulative distribution function of the first category in the response scale, as in Equation 4:

$$TFR_x = max\left(0, \frac{E(x) - E(1)}{1 - E(1)}\right) \tag{4}$$

Causation, necessity and (robust) sufficiency

Burke R. Johnson, Federica Russo and Judith Schoonenboom (2019) provide an overview of the causal interpretation in mixed methods research, from its philosophical roots (Hume, 1999; Mackie, 1974; von Wright, 1975) to modern probabilistic and statistical modeling of causation. All major thinkers agree that causal analysis in the social sciences is a difficult endeavor. For some obvious events it is easy to identify what people normally refer to as causes, but it is not exactly straightforward for events with a more complex structure.

It is, however, possible to identify a set of causal factors that are associated with an outcome, by identifying those that act as difference-makers; that is, the formation of such a set makes a qualitative difference on the outcome. Since no exact relation can be observed between a single cause and a single outcome, philosophy and social science methodology provide a good replacement in the concepts of sufficiency and necessity

Sufficiency and difference-making theories are compatible, both stating that for the outcome to occur the cause needs to occur as well (in this context, a “cause” is almost never a single cause but rather a conjunction of multiple causal factors, both in their presence and absence). When a cause X is sufficient for the outcome Y, then Y is always present when X occurs, and X does not occur in the absence of Y. In formal notation, sufficiency is denoted by: $X \Rightarrow Y$

Necessity is a mirrored concept, in that when X is a necessary condition for an outcome Y, then X is always present when Y occurs and Y does not occur in the absence of X. In formal notation, necessity is denoted by: $X \Leftarrow Y$.

In many situations, a causal condition can be sufficient without being necessary. This is the very definition of equifinality, when the outcome is produced in many different ways. In the other direction, a causal condition can be necessary but not sufficient, which is the essence of John Leslie Mackie’s philosophy of causation and his well-known INUS: insufficient but necessary part of an unnecessary but sufficient condition. An INUS condition is therefore part of a sufficient conjunction, which is itself one way among many other to instantiate the outcome.

In set theory, sufficiency and necessity can be expressed as a subset / superset relations: if a causal condition is sufficient, it is a subset of the outcome, and if a causal condition is necessary, it is a superset of the outcome. That the outcome Y needs to be a subset of the causal condition can be counter-intuitive, but it makes sense when considering what necessity logically means. For a fire to happen, heat is a necessary condition (in the absence of which fire would not start) but a fire cannot happen only in the presence of heat.

Other INUS conditions are conjunctively needed (like oxygen and a spark), but the fact is heat is omnipresent in all situations where a fire is started. Heat is a superset of the outcome fire, given there are many situations where there is heat without a fire. The other way round, whenever there is fire heat is present too, which makes the fire outcome a subset of the heat set.

The subset / superset relations pave the way for another two important concepts in QCA:

- inclusion (which Ragin calls *consistency*), and
- coverage.

Inclusion should be self explanatory, it shows the proportion of a set X that is included into another set Y. It is calculated as the set intersection between the two sets X and Y, divided by the entire space of the set X.

$$inclS_{X \Rightarrow Y} = \frac{\sum min(X, Y)}{\sum X} \tag{5}$$

Figure 3 shows an almost complete inclusion of the causal set X into the outcome set Y, using a

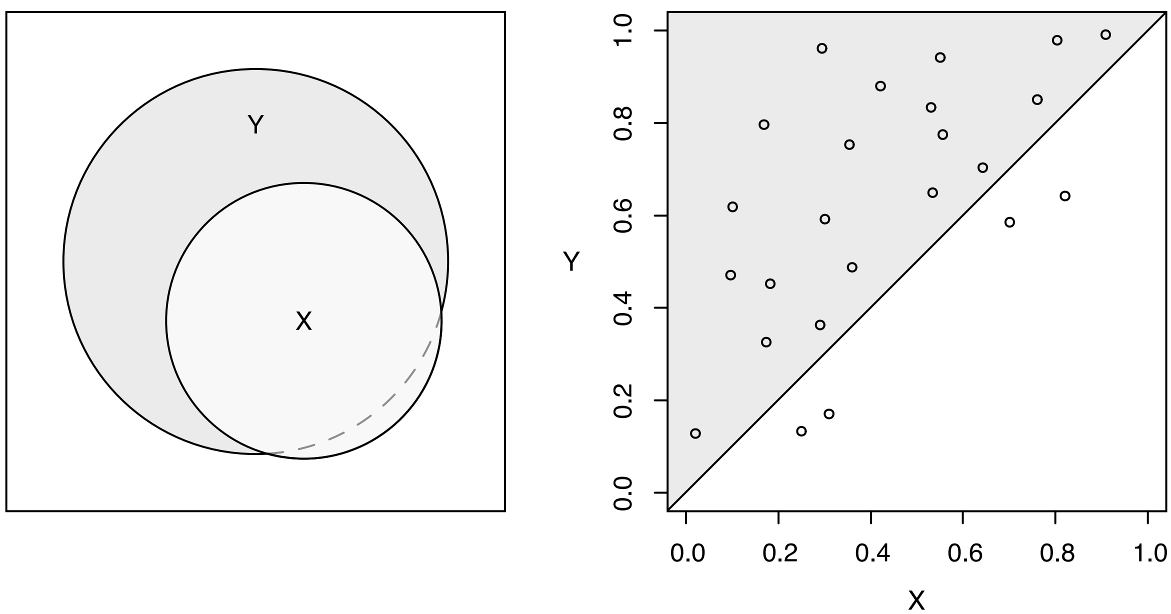

Figure 3: Almost complete inclusion of X into Y

Venn/Euler diagram in the left side and what is called a fuzzy XY plot in the right side. They are both typical diagrams to signal (an almost perfect) sufficiency, the reason for which it is written as *inclS* in Equation 5. Distinctively in the XY plot, almost all points are located above the main diagonal.

Coverage is the proportion of space from the outcome set Y that is covered by the set X. The more space is covered by X, the more causally relevant is X for Y (vaguely similar to the coefficient of determination R^2 from the linear regression).

$$covS_{X \Rightarrow Y} = \frac{\sum min(X, Y)}{\sum Y} \tag{6}$$

Necessity and sufficiency are twin concepts, inclusion for sufficiency having the same formula as the coverage for necessity, and the coverage for sufficiency is the same as the inclusion for necessity. In his contributions to the analysis of necessity, Gary Goertz (2006) differentiates between relevance and trivialness (for instance, air is a trivial condition for a fire, since it is omnipresent), proposing a measure of trivialness that was fine-tuned by Carsten Q. Schneider and Claudius Wagemann (2012) in their relevance of necessity measure:

$$RoN = \frac{\sum(1 - X)}{\sum(1 - min(X, Y))} \tag{7}$$

The relevance of necessity should not be confused with the concept of "causal relevance", which is related to sufficiency.The point of qualitative comparative analysis is to apply the Boolean minimization algorithm in order to eliminate as many irrelevant conditions as possible, preserving only those conditions that are causally relevant. The surviving prime implicants (conjunctions of causal conditions) are always sufficient for the presence of the outcome.

Table 5: Sufficiency in material implication (T = true, F = false)

X	Y	$X \Rightarrow Y$
F	F	T
F	T	T
T	F	F
T	T	T

Boolean minimization sufficiency is different from the sufficiency concept defined in formal propositional logic from Table 5. Material implication is also a logical form of sufficiency, namely that anything is sufficient for anything as long as there is no counter proof. The sufficiency statement $X \Rightarrow Y$ holds true for all possible combinations between a cause X and an outcome Y, except for the third row where the outcome does not happen (it is false), while the cause is present (it is true).

In propositional logic, that is the only possible way to invalidate a sufficiency statement. This property has an application in causal analysis, in that any causal condition that passes the test of material implication is bound to be causally relevant. However, that is not a guarantee the cause is atomically sufficient, does not guarantee the outcome happens only in the presence of causal condition X alone (other causal conditions might conjunctively be needed to make the outcome happen).

This is the reason why a distinction must be made between the concept of "sufficiency" that is common in both causal analysis and material implication), and "robust sufficiency" which is the type of sufficiency that passes the test of Boolean minimization and guarantees the outcome always happens in their presence.

The quest for causal relevance is a double effort involving parsimony (Baumgartner, 2015) but also robust sufficiency. Favoring parsimony at any cost runs in the risk of losing robust sufficiency, which might not always be optimally parsimonious (it might contain irrelevant conditions that are not causal), but on the other hand the outcome is guaranteed to happen.

Constructing the truth table

The analysis of necessity and the analysis of sufficiency are key methodological tools in QCA. For the analysis of sufficiency, following the introduction from the first section, researchers need to construct a truth table that contains all possible combinations of presence and absence of the causal conditions introduced in the analysis.

The configurations of the observed cases are allocated to specific rows from those rows in the truth table to which they resemble most. For crisp sets that is extremely simple, the observed configurations having exactly the same values as the corresponding truth table configurations.

Table 6 contains all possible configurations for the values of two binary crisp conditions A and B. As both of them are binary crisp (each having two values 0 and 1) there are four possible configurations: both absent, one of them present and the other not, and both present. For binary crisp conditions (the most common analytic scenario), the total number of truth table configurations is equal to 2^k, where k is the number of causal conditions.

Table 6: Truth table configurations for two causal conditions A and B

A	B	OUT
0	0	
0	1	
1	0	
1	1	

A more general and equivalent formula for the number of rows in a truth table, that covers multi-value conditions as well as binary crisp, is the product of the number of levels l for all k causal conditions as given in Equation 8:

$$\prod_{c=1}^{k} l_c = l_1 \times l_2 \times \cdots \times l_k \tag{8}$$

The output column OUT will contain the value of the outcome (either presence or absence), function of the cases that are allocated to each row of the truth table. The allocation process is relatively simple for crisp data, but fuzzy sets are more challenging because they tend to resemble multiple configurations.

On one hand, the Boolean minimization algorithm works precisely because the input data is Boolean, with an exponential but still finite number of possible configurations. On the other hand, fuzzy sets have an infinite number of possible values, and no algorithm can deal with that kind of information. The procedure to reduce the fine grained fuzzy sets to a finite, crisp equivalent in the truth table was introduced by Ragin (2008).

Ragin used the traditional idea that truth table configurations are the corners of a multi-dimensional vector space, and the combination of fuzzy values specific to each case is more or less close to one of these corners. They can be located in the immediate proximity of many corners (especially when some of the set membership scored are near the point of maximum ambiguity), but it is only one corner they are closest to. In a similar interpretation, no real-life case is ever perfect but all cases resemble a certain ideal type.

Figure 4 illustrates the simplest possible examples of a case being closest to the corner “10” (first condition present, second absent) in a bidimensional vector space corresponding to two causal conditions, and respectively closest to the corner “100” in a three-dimensional vector space. To determine exactly at which corner the cases belong to, Ragin’s procedure involves calculating an inclusion score for all possible pairs of cases and corners, in a matrix having all cases on the rows and all truth table rows on the columns.

This procedure is simple for a small number of conditions, but as more are introduced the number of pairs becomes larger and larger. An improved method was introduced by Dușa (2019) with only two steps:

- determining at which causal conditions the cases have an inclusion score above 0.5 to obtain a vector of 0s and 1s, and
- calculating its Hadardmard product with a vector formed by the cumulative product of the number of levels for all causal conditions except the last, and starting with 1.

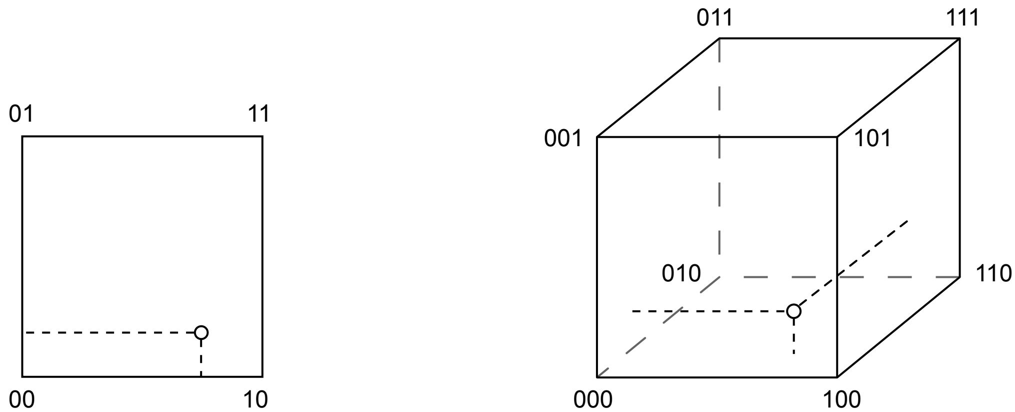

Figure 4: Vector space for two conditions (left) and three conditions (right)

Once the cases are allocated to the truth table configurations, Ragin's procedure has to determine the value of the OUT column, that is a function of the consistency score with the outcome, calculated from the causal conditions data corresponding to the corners. For instance, a corner such as 100 is equivalent to a conjunction of three causal conditions $A{\sim}B{\sim}C$, where the conditions B and C are negated. The consistency score for this corner is calculated between the intersection (taking the minima) of the transformed data, against the outcome.

If the value of this consistency (inclusion) score is greater than a certain threshold that is determined by the researcher (good practice advice suggest no less than 0.7, preferably at least 0.8) the OUTput column is coded as 1, otherwise 0. In this way, all observed cases participate in calculating the consistency score with each of the rows in the truth table that have at least one allocated case.

When the number of causal conditions is very large, most of the rows in the truth table will have no cases allocated. That happens because of the phenomenon called *limited diversity* which is especially specific to the social sciences. The reality we observe is not random and therefore does not cover all possible, conceivable causal scenarios. Quite the contrary it follows certain patterns, which means that more observed cases do not necessarily contribute to more diversity but simply cluster together in the same corners.

This is a strength of the Qualitative Comparative Analysis, where the emphasis is less on the number of cases (as in statistics) but rather on the number of observed causal configurations. It can operate with a medium N as well as with a large N, and after a certain threshold the number of cases becomes almost irrelevant because of the limited diversity, when additional cases will only pile up in the already identified causal configurations. Truly important is to observe all causal configurations that do exist, for any one that is missed can lead to potentially incomplete or sometimes even erroneous conclusions.

Table 7 presents the truth table for the fuzzy version of Seymour Martin Lipset's (1959) data, using a 0.7 inclusion cut-off, where DEV means level of development, URB is the level of urbanization, LIT the level of literacy, IND the level of industrialization and STB is government stability. Lipset's indicators for the survival of democracy (outcome, denoted by SURV in the original data) during the inter-war period is a well-known data set in political science and has been used as example data in most chapters in Benoit Rixoux and Ragin's *Configurational Comparative Methods. Qualitative*

Comparative Analysis (QCA) and Related Techniques (2009).

Table 7: The truth table for the Lipset data

	DEV	URB	LIT	IND	STB	OUT	n	incl	PRI	cases
32	1	1	1	1	1	1	4	0.904	0.886	BE,CZ,NL,UK
22	1	0	1	0	1	1	2	0.804	0.719	FI,IE
24	1	0	1	1	1	1	2	0.709	0.634	FR,SE
6	0	0	1	0	1	0	1	0.529	0.228	EE
5	0	0	1	0	0	0	2	0.521	0.113	HU,PL
31	1	1	1	1	0	0	1	0.445	0.050	DE
23	1	0	1	1	0	0	1	0.378	0.040	AU
2	0	0	0	0	1	0	2	0.278	0.000	IT,RO
1	0	0	0	0	0	0	3	0.216	0.000	GR,PT,ES
3	0	0	0	1	0	?	0	-	-	
4	0	0	0	1	1	?	0	-	-	
7	0	0	1	1	0	?	0	-	-	
8	0	0	1	1	1	?	0	-	-	
9	0	1	0	0	0	?	0	-	-	
10	0	1	0	0	1	?	0	-	-	
11	0	1	0	1	0	?	0	-	-	
12	0	1	0	1	1	?	0	-	-	
13	0	1	1	0	0	?	0	-	-	
14	0	1	1	0	1	?	0	-	-	
15	0	1	1	1	0	?	0	-	-	
16	0	1	1	1	1	?	0	-	-	
17	1	0	0	0	0	?	0	-	-	
18	1	0	0	0	1	?	0	-	-	
19	1	0	0	1	0	?	0	-	-	
20	1	0	0	1	1	?	0	-	-	
21	1	0	1	0	0	?	0	-	-	
25	1	1	0	0	0	?	0	-	-	
26	1	1	0	0	1	?	0	-	-	
27	1	1	0	1	0	?	0	-	-	
28	1	1	0	1	1	?	0	-	-	
29	1	1	1	0	0	?	0	-	-	
30	1	1	1	0	1	?	0	-	-	

As can be seen, most of the configurations on the rows are empty and have a question mark in the OUT column, indicating the limited diversity phenomenon. As such, the truth table contains three categories of configurations:

- the positive configurations, those which have been allocated at least one case and the output was determined equal to 1 (outcome is present at the top, with a dark grey background)
- the negative configurations, those which have been allocated at least one case and the output was determined equal to 0 (outcome is absent, with a light grey background)
- the so-called *remainders*, those which lack empirical evidence (have no observed cases allo-

cated) making it impossible to determine a value for the output column

Deriving the solutions

Once the truth table is constructed, the next stage is set to perform the Boolean minimization process, with the explicit intent to eliminate the irrelevant conditions and obtain a causal recipe where all surviving conditions make a difference on the outcome. This involves a series of steps and a sequential dialogue with the data, starting with the classical complex solution (QCA-CS), then gradually introducing counterfactual assumptions to further refine it.

QCA-CS strictly uses the observed, positive causal configurations from the truth table. Depending on the number of causal conditions and on the number of observed configurations, this type of solution can be more or less simplified but usually not very much. Sometimes, if the observed configurations are very different they cannot be minimized at all and QCA-CS is identical to the initial positive observed configurations.

In the Lipset truth table, there are three positive observed configurations: DEV·URB·LIT·IND·STB, DEV·∼URB·LIT·∼IND·STB, and DEV·∼URB·LIT·IND·STB. Following the Boolean minimization principles:

- DEV·URB·LIT·IND·STB and DEV·∼URB·LIT·IND·STB minimize into DEV·LIT·IND·STB (URB being irrelevant), while
- DEV·∼URB·LIT·∼IND·STB and DEV·∼URB·LIT·IND·STB minimize into DEV·∼URB·LIT·STB (this time IND being irrelevant).

Neither of these last terms can be further minimized, therefore QCA-CS is their disjunction, or logical union:

$$\text{DEV}\cdot\text{LIT}\cdot\text{IND}\cdot\text{STB} + \text{DEV}\cdot\sim\text{URB}\cdot\text{LIT}\cdot\text{STB} \Rightarrow \text{SURV}$$

It is also called the *conservative* solution, because it makes no counterfactual assumptions on the remaining unobserved configurations. These remainders play a central role in the Boolean minimization algorithm, allowing researchers to make inferences beyond the observed empirical data. In the absence of pure experimental data, counterfactual analysis is actually very common in the social sciences (Mahoney and Barrenechea, 2017; Morgan and Winship, 2015), with an example from David Hume (1999):

> we may define a cause to be an object followed by another, and where all the objects, similar to the first, are followed by objects similar to the second [...] Or, in other words, where, if the first object had not been, the second never had existed (p.37)

Expressions such as "if X were true, then Y would happen", where the condition X is not empirically observed, is a typical counterfactual statement. In order to further minimize the conservative solution, the overall strategy is to counterfactually analyze the remainders. Assuming that everything that is unobserved (all remainders) would lead to the presence of the outcome, is the most aggressive form of minimization leading to what is called the *parsimonious* solution (QCA-PS) which has the smallest number of relevant causal conditions.

Concretely, QCA-PS involves all truth table configurations where the OUT column is not equal to 0 (the positive ones and the remainders), and iteratively minimize until all irrelevant conditions are eliminated. The following solution is thus produced:

$$\text{DEV}\cdot\text{STB} \Rightarrow \text{SURV}$$

Both DEV and STB are surely relevant causal conditions, and this compact solution seems to offer a more attractive and clear perspective over what determines the survival of democracy. In reality, it is highly questionable if some of the remainders can truly be associated with the presence of the outcome.

Table 8 presents a hypothetical dataset with the relation between pregnancy, being a female or a male and the outcome of extremely safe driving, with the hypothesis that pregnant females drive extremely safe. There are three observed cases, and only in the first the outcome is present. Since all causal conditions are measured in binary crisp sets, the truth table corresponding to these data is exactly the same, with a single remainder on the fourth row.

The conservative solution for these data corresponds to our hypothetical expectation that P·F ⇒ ESD, whereas QCA-PS shows that pregnancy alone is sufficient to explain the extremely safe driving: P ⇒ ESD. While pregnancy is indeed causally relevant, as it happens QCA-PS made use of the fourth row (the unobserved remainder) which happens to be an impossible configuration because it represents a pregnant male.

Faced with such situations, Ragin and John Sonnett (2005) developed a procedure that prevents some of the remainders from being used in the minimization process, introducing the so-called *directional expectations*, resulting in a third type of solution called *intermediate* (QCA-IS), that is located in between the complexity space having QCA-CS and QCA-PS at the extremes. The entire process is known as the Standard Analysis (SA), which was later extended by Schneider and Wagemann (2013) into the Enhanced Standard Analysis (ESA), that further eliminates remainders that are impossible, difficult, contradictory or untenable.

QCA-CS and QCA-PS are not just the opposite ends on the complexity continuum, they are both some sort of "ideal" types of solutions. One of them strives for robust sufficiency but sometimes allows non-causal conditions in the solution, while the other strives for parsimony and strictly preserves only the causally relevant conditions but it sometimes sacrifices the robust sufficiency.

The ideal solution is the most parsimoniously possible, robustly sufficient solution. Both types of solutions are desirable, despite the fact they act in opposite directions. QCA-IS seems to be the one that satisfies both goals, but requires the researcher to make informed and well documented decisions about what remainders to include, as well as what remainders to exclude from the Boolean minimization process.

Table 8: Relation between pregnancy (P), females (F) and extremely safe driving (ESD)

P	F	ESD
1	1	1
0	0	0
0	1	0
1	0	?

QCA compared to regression

There seem to be high entry barriers for a new research methodology, just like a new competitor in a market. It was to be expected that an alternative methodology would raise a lot of criticism, especially from quantitative researchers, but as it turns out most of that criticism is due to an expectation that QCA would function in a similar way to a regression model.

Despite presenting some similarities, in that both QCA and the regression techniques propose explanatory models with a number of "causal conditions" (in QCA) or "independent variables" (in regression) plus an "outcome" or a "dependent variable", their analytic procedures are different.

Numerous papers already considered using QCA versus the regression analysis, comparing the two (Grofman & Schneider, 2009; Katz, vom Hau & Mahoney, 2005; Seawright, 2005), using moderate sample sizes but also large samples as well (Schneider & Makszin, 2014; Vis, 2012), triangulating (Skaaning, 2007) or integrating them (Fiss, Sharapov & Cronqvist, 2013), and even combining the two into a multi-method approach (Stolz, 2015).

What is clearly common in most of the critical papers is an attempt to evaluate the set theoretic comparative methods (STCM) as if they were answering the same fundamental questions that are presented in the quantitative research, in the spirit of Gary King, Robert O. Keohane and Sidney Verba's *Designing Social Inquiry* (1994) and its subsequent continuation *Rethinking Social Inquiry: Diverse Tools, Shared Standards* from Henry E. Brady and David Collier (2010).

James Mahoney (2010) does a thorough forensic analysis on both these classical resources for quantitative analysis, and reveals one of the main differences between the regression analysis and QCA, since researchers:

> … distinguish between approaches that seek to estimate the average effect of particular independent variables (that is, effects of causes) from those that attempt to explain why specific cases have particular outcomes (that is, causes of effects)… (p.132)

The difference between the effects of causes (EoC) and causes of effects (CoE) is also treated by Judea Pearl (2015), but from a very different perspective involving conditional probabilities, to claim that individual cases can be counterfactually determined from statistical data. As interesting as it is, such a framework still requires a prior large sample of experimental data before calculating such probabilities, which is not always the situation in social and political sciences, a problem leading to another potential difference between applying QCA or regression techniques: sample size.

A typical situation in macro-comparative research involves a dataset containing a small number of cases (most often, countries). The obvious advice coming from quantitative research is to collect large samples from each country that would facilitate a proper statistical analysis, but there are situations when countries are compared only for aggregated measures such as GDP, a proxy for the level of development. If studying a particular group of countries (e.g. in a particular geographical region), the sample size can be anywhere between 2 and 15, which is not nearly enough for a statistical analysis due to the very small and rapidly decreasing number of degrees of freedom (Mahoney, 2010; Rubinson & Ragin, 2007).

Regression techniques focus on the average net effects of competing independent variables, controlling for all other variables in the model, while QCA focuses on cases understood as configurations of causal conditions. This is another difference between the two frameworks, the first being variable oriented and the second being case oriented. Consequently, their analytic and mathematical procedures are different, as also observed by Thiem, Michael Baumgartner and Damien Bol (2016).

For a quantitative researcher trained in the linear algebra tradition, the equation $A + 1 = 1$ would be very peculiar. But that makes perfect sense from a set theoretical point of view using Boolean algebra, where the "+" sign (unlike the linear algebra where it means an addition) signals a logical disjunction, or a set union, and the number 1 signals the entire universe. The union between the universe and anything (A) is still the universe, given that A is part of the universe anyways. A similar line of thought explains why the equation $A * A = A$ holds in the Boolean algebra (where the sign "*" stands for a set intersection, a logical conjunction), whereas in linear algebra it is equal to A^2 (given the same sign involves a multiplication).

The quantitative tradition set by King, Keohane and Verba (1994) largely ignores set theory where cases are compared against each other, and favors a statistical framework where values are compared against the measures of central tendency such as the mean or the median. Both frameworks use the terms *high* and *low*, but whereas linear algebra identifies values that are higher or lower than the average, in Boolean algebra a value is high only through a reference to an established theoretical standard, as described in the section about calibration. Defining sets and attributing set membership scores is tightly linked to the process of concept formation, which is largely missing in the quantitative approach.

This has a direct influence over how researchers relate to their data. It would not be possible for the quantitative approach to generate theories, but only to test theories using inferential statistics. In QCA and general qualitative approaches, cases are compared to identify configurational patterns that would help in advancing a theory about an outcome of interest.

For purposes of causal analysis, neither approach has a definitive answer. Just as correlations do not entail anything about causation, patterns of association are not a guarantee that configurations are causal. However, it is likely that set theoretic methods are closer to achieving the objective of identifying causes, since their purpose is to detect regularities in the data. Researchers may never know what the true causal model really is, but at least it is possible to assess whether hypothesized causal conditions are necessary, or sufficient, or more often if they are conjunctive INUS conditions as defined by Mackie (1974).

Neither framework is fail proof at the problem of omitted variables, as shown by Claudio M. Radaelli and Claudius Wagemann (2018). Researchers may never be certain their theoretical models are completely specified, or that they have correctly specified the set of control variables. In the regression analysis, some of these problems can be detected by inspecting the residual plots: if a relevant variable is omitted from the model, its effect is transferred to the residuals which correlate with the independent variables. On the other side in QCA, mid-sized raw consistency values could be an indicator of omitted causal conditions, just as the presence of contradictory cases located in the lower-right part of the XY plots.

While regression searches for "the" perfect model, perhaps another way to approach the research situation is to ask whether there are multiple, equally valid models that explain a certain outcome. This is a sensible approach if considering causal complexity and equifinality, that postulates the same outcome can be achieved via many causal paths, sometimes using different sets of causal conditions.

Configurational patterns are commonly considered to be similar to interaction effects in the regression analysis, but Tammo Straatmann and colleagues (2017) show that interaction effects in regression are not always significant even when QCA points to relevant configurations. The lack of significance for a regression coefficient does not necessarily mean there is no relationship between the independent variable and the dependent variable, but rather the relationship might not be symmetric and additive.

That is yet another difference between the two frameworks: while regression (involving correlations) are defined by causal symmetry, set relations are defined through causal asymmetry: there are different explanations for how an outcome occurs, versus how an outcome does not occur.

Finally, regression models are applicable only in those situations where the underlying assumptions are met. If the scatterplot is not linear, there can be no linear regression. If the variables are not normally distributed, and for multiple regression if multivariate normality is not met, the regression model cannot be applied. The list of assumptions is quite large (e.g. no multicollinearity, no heteroskedasticity, no autocorrelation), whereas QCA models are indifferent of such assumptions.

While geared towards revealing the differences between the two research frameworks, the purpose of this section is not to show that one outperforms the other but rather to indicate there are situations where either one might be more appropriate depending on the research question, or the research scope, or sometimes on the sample size.

Conclusion

Part of the Configurational Comparative Methods, QCA is a relatively young but already robust methodology, witnessing an explosion of theoretical developments (Beach, 2018; Dul, 2016; Garcia-Castro & Ariño, 2016; Oană & Schneider, 2018; Rohlfing & Schneider, 2018). It is ideally placed between the classical quantitative, statistical tradition and the qualitative world, to provide answers to questions that none of those can provide.

It would perhaps be suitable to end this entry by reiterating that unlike correlation-based analyses, QCA models causal complexity and has the distinctive features of asymmetrical causation and equifinality. The solutions in QCA are conjunctural, meaning the outcome happens only at the intersection of some non-redundant causal conditions.

The causal asymmetry feature is particularly interesting, in that analyzing the causes that make an outcome happen is something very different from analyzing the causes that lead to the absence of the outcome. Constructing the truth table for the absence of the outcome follows the same procedure, but inclusion scores in the negation of the outcome (usually calculated by subtracting the fuzzy membership scores from 1) are not simply the inverse of the inclusion scores for the presence of the outcome.

It is arguably less intuitive, but entirely possible for a case to have high inclusion scores in both the presence and the absence of the outcome, and quite common two inclusion scores do not sum up exactly to 1. This should be a methodological signal for those researchers who prefer to measure indicators using a bipolar scale, where the negative pole and the positive pole might very well be two entirely different fuzzy sets, and empirical cases can have inclusion scores in both. The comparative methods in general, and the Boolean algebra in particular, unravel a promising methodological toolkit.

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